Research Note RM-518

February 1993

USDA Forest Service

Rocky Mountain Forest and Range Experiment Station

Formulating MAXMIN Objectives in National Forest Planning Models[1]

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MAXMIN linear programming objective functions are useful in national forest planning models for optimizing management strategies for critical resources. This paper describes the MAXMIN approach to analyzing forest management problems and presents mathematics for formulating MAXMIN objective functions. An example using the USDA Forest Service FORPLAN modeling system is provided.

Keywords: MAXMIN, forest planning, linear programming, FORPLAN

Introduction

National forest planners are searching for ways to address management concerns regarding the reduction of forest resources to critically low levels. When the principal objective in an analysis is to find a schedule of forest practices that raises the lowest level of expected production for critical resources as high as possible, the MAXMIN approach is proving very useful. This paper briefly describes the MAXMIN approach to analyzing forest management problems. It also demonstrates how to implement this approach in FORPLAN (Johnson et al. 1986), the current USDA Forest Service planning system. Appendix A provides a specific example of how to incorporate a MAXMIN objective function in a FORPLAN data set. Appendix B shows the linear programming matrix tableau for that model.

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The MAXMIN Approach

MAXMIN objective functions are an approach for maximizing the minimum achievement among a set of products, applied here in linear programming models. The mathematical development of MAXMIN linear programming arose from applications of fuzzy set and systems theory initially advocated by Zadeh (1965). Bellman and Zadeh (1970) were instrumental in introducing these concepts to the management sciences. Development of fuzzy mathematical programming systems quickly followed (Tanaka et al. 1974; Zimmermann 1976). A concise review is available from Ignizio (1982), and a number of textbooks now provide detailed coverage of the subject (Terano et al. 1992 among others).

Researchers in renewable natural resource disciplines have produced a number of MAXMIN applications in recent years. Hof, Pickens, and Bartlett (1986) developed a MAXMIN approach to timber harvest scheduling where the minimum periodic harvest among 15 time periods was maximized as an alternative to using traditional nondeclining yield constraints. Bare and Mendoza (1992) similarly applied a MAXMIN formulation to deviations from nondeclining

yield. In these types of applications the MAXMIN objective finds an equity-oriented solution in the Rawlsian (1971) sense in which the least well-off individual (time period) is optimized. This principle can be applied to any number of natural resource problems (fig. 1), including analysis of forest health or species diversity (Hof and Raphael, in press).

Mendoza and Sprouse (1989) discuss MAXMIN applications involving incommensurate resource management objectives. While the MAXMIN approach does not solve the problem of valuation of incommensurable outputs, useful solutions can be generated that probably would not be considered otherwise. Extending analysis of resource management objectives further, Hof and Pickens (1991) and Hof, Kent, and Pickens (1992) have applied MAXMIN methods to estimate the probabilities of output targets being met.

MAXMIN objectives have also been used to optimize systems subject to limiting factors. For example, Pickens, Hof, and Bartlett (1987) showed how to use MAXMIN objectives to select range management prescriptions that maximize total animal unit months of grazing by maximizing the limiting livestock nutrient. These and other natural resource applications are discussed in detail by Hof (1992).

Mathematical Formulation in FORPLAN

A typical MAXMIN formulation involves setting up the problem around a special variable, usually referred to as λ. The problem, then, is

maximize

$$\lambda$$

subject to

$$\lambda \leq Q_i \quad \forall\, i$$

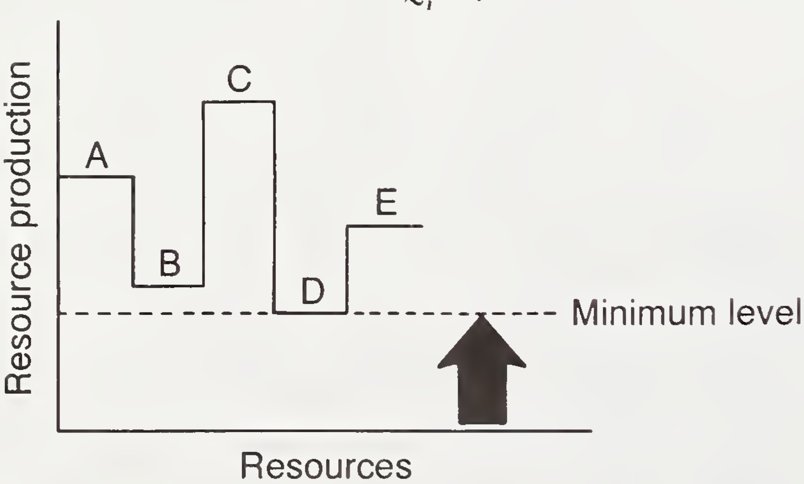

Figure 1.—A MAXMIN objective is applied to raise the lowest resource production level as high as possible.

where Q_i are the product quantities of concern, such as critical livestock nutrients. These quantities are represented in FORPLAN matrices by A-matrix coefficients of the land management prescription variables rather than as explicit product decision variables (Johnson and Stuart 1987). Replacing the Q_i accordingly, the constraint set becomes,

$$\lambda \leq \sum_{j=1}^{n} q_{ij} X_j \quad \forall\, i$$

where q_{ij} are production coefficients for each of the n land management decision variables, X_j. While FORPLAN land management decision variables are more correctly enumerated as timing choices across management prescriptions across analysis areas, X_j will serve adequately here.

Similarly, a decision variable must be generated for λ. This is somewhat awkward in FORPLAN since the usual method of generating decision variables is to define them as land management prescriptions with yields for a variety of outputs. The approach taken here, then, is to define λ as an output from a dummy "land prescription" subject to a dummy "acreage" constraint. In order to prevent this constraint from limiting λ maximization, an arbitrarily large number, M, is used for the right-hand side, resulting in

$$\lambda = kD$$
$$D \leq M$$

where D is the dummy land prescription decision variable (see prescription "MXMN" in appendix A) and k is the dummy production coefficient for λ. These equations can be reduced by noting that

$$\lambda\ / k = D \leq M$$

which becomes

$$\lambda \leq kM.$$

This equation clarifies that in FORPLAN the mathematical product kM is actually the quantity that must be scaled large enough to be nonbinding when maximizing λ. When k is set to 1, the "acreage" assigned to λ production is identical with λ production. Otherwise, k can be used to help scale λ relative to M.

Since acreage constraints in FORPLAN are formulated as equality constraints, an explicit slack variable with no value in the model must be added. Fortunately, FORPLAN is designed to build just such a variable whenever yields are

not assigned (see prescription "SLAK" in appendix A). The "acreage" inequality above becomes

$$D + S = M$$

where S is the new slack "acreage" variable.

Completing the transition to FORPLAN system structure requires one more step. Constraints must be imposed to ensure that resource production is always at least equal to λ or some positive multiple of it. Absolute constraints provide one approach. Aggregate accounting variables can be used in FORPLAN to build absolute constraints on the difference between two resource outputs. Another approach, taken here, is to use general relational (proportional) constraints. This formulation places the amount of λ in one constraint row and the $q_{ij}X_j$'s in additional rows. The two constraint sets are then tied together with a transfer column, T, expressing the proportional limits between them:

$$\lambda - T = 0$$

$$\sum_{j=1}^{n} q_{ij}X_j - cT \geq 0 \quad \forall\ i.$$

These constraints require that the outputs of concern exceed some constant times λ in all cases. As long as c is a constant greater than zero, maximizing λ will provide a MAXMIN solution for equally weighted resource production. FORPLAN provides a simple input section for constructing these constraints.

While equal weighting as portrayed here is typical, there are cases where unequal weighting of the various products is necessary. For example, in a MAXMIN nutrient supply problem it may be known a priori that two nutrients must be available in a 2:1 ratio to be fully utilized. This requirement would then be formulated by replacing the constant, c, with a set of coefficients, c_i. In the 2:1 nutrient example the coefficients could be $c_1 = 2$ and $c_2 = 1$, or some positive multiple of these.

Optimizations involving multiple time periods, for example a MAXMIN periodic timber harvest problem, are readily constructed by maximize

$$\lambda$$

subject to

$$\sum_{j=1}^{n} q_{hij}X_j - c_iT \geq 0 \quad \forall\ h,i$$

$$\lambda - T = 0$$

$$\lambda - kD = 0$$

$$D + S = M$$

where there are h time periods and all other symbols are as previously defined. The q_{hij}'s are now coefficients of the j = 1 to n prescription timing choices representing production quantities per unit of land for each of the i critical resources in each of h time periods.

Of course, acreage constraints for the land management decision variables, X_j, would be needed to complete the linear program. These are inherent in the FORPLAN system even though they have not been shown here. The FORPLAN data set in appendix A lists the input for generating a model of the above form to maximize minimum periodic timber production over 4 (h = 1,4) decades. The model has one resource output (i = 1,1), timber, produced by 5 of 6 (n = 6) prescription timing choices from a single analysis area.

Operational Considerations and Conclusions

Early experiments with MAXMIN formulations in natural resource optimization models were fraught with computational difficulties (Pickens and Hof 1991). Similar difficulties have occurred with FORPLAN models in the past when the objective function row was sparse relative to the number of decision variables in the problem. For example, computational problems often occurred when demand curves were used or when first decade timber harvest was maximized subject to nondeclining yield constraints. Recent improvements in linear program solvers appear to have greatly alleviated these computational problems, although solution times can still be somewhat slow. Three operational considerations should be kept in mind with MAXMIN models, however.

First, all linear programming models can be sensitive to poor scaling in the matrix, and MAXMIN models are no exception. Three sets of constants (c_i, k, and M) are available for scaling the MAXMIN output, λ, used for the objective function in the system of FORPLAN equations described in this paper. While it is essential that λ be "unconstrained" by the mathematical product, kM, excessively large values for k and M can lead to solution difficulties. The values for these constants, as well as for c_i, should be carefully scaled to the rest of the model. As an extreme example to highlight how one would go about MAXMIN scaling in FORPLAN, suppose that any

product, Q_i, might approach but not reach a value of 1000. Also suppose that other coefficients and bounds in the model are already between 0.0001 and 10. To keep the model reasonably well-scaled, we might wish to set values of c, k, and M at 10. These values would allow λ to get as large as kM, or 100. Additionally, having c set to 10 requires that all Q_i be at least 10 times as large as λ. Thus, increases in λ will push up the quantity of all products by a factor of 10 as long as they do not reach the limit of 1000.

The second operational consideration is matrix size. When MAXMIN objectives are included in a FORPLAN model, a number of general relational constraints or absolute constraints (not shown in this paper) and dummy analysis areas are required to make these objective functions work properly. Often this will have little impact on matrix size since the MAXMIN objective will be used in place of other constraints. The sample FORPLAN data set in appendix A is a good example; the MAXMIN objective has been used to replace nondeclining yield constraints on timber harvest. Whenever MAXMIN objectives are included in FORPLAN models in addition to existing constraint sets, however, analysts should be prepared to cope with substantially larger matrices.

Finally, analysts should bear in mind that MAXMIN formulations are a form of goal programming. They can and often do produce inferior solutions from a mathematical point of view. Although the limiting factor (or factors) in the model will be maximized, improvements can sometimes still be made to the remaining (nonlimiting) products. In some cases this may be irrelevant, such as when maximizing the production of limiting livestock nutrients. Here, any additional production of the surplus nutrients may be of no value. In other cases, initial MAXMIN solutions can be suboptimal since producing additional amounts of the nonlimiting resources may be of value. For example, maximizing the habitat effectiveness for the most limiting species should not preclude a search for alternate optima that may improve habitat effectiveness for other species as well. Successive optimizations in a rollover fashion (Kent et al., in press) may be required to first maximize the production level of the most limiting resource with successive optimizations to drive remaining resources to their joint production frontier.

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Appendix A

An Annotated Sample FORPLAN Data Set

TITLE N N N N N

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FORPLAN Release 14.2: MAXMIN Objective Function Example

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Data set developed by Mike Bevers, RMF&RES, and Bruce Meneghin, LMP Systems,

USDA Forest Service, Fort Collins, Colorado

August 1992

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This data set shows how to incorporate a MAXMIN objective function in FORPLAN. More explanation and additional sources of information on uses of MAXMIN objectives can be found on previous pages of this Research Note. Refer to the FORPLAN Users Guide (USDA Forest Service, WO-LMP) for more information regarding FORPLAN input data conventions and interpretation.

The data set has been derived by paring down the third Brush Mountain example used for basic FORPLAN training by the Fort Collins WO-LMP Systems Section. For simplicity, and to highlight the input requirements for a MAXMIN objective, the original model has been reduced to a single analysis area and a single timber harvest prescription with a number of harvest timing choices. All unessential activities, outputs, and constraints have been removed to reduce clutter for the reader. Timber sustainability constraints have been retained to demonstrate how these might be changed under a MAXMIN objective.

This example employs a formulation designed to maximize the smallest periodic volume of timber harvested throughout the planning horizon. A similar formulation could as easily be used to maximize the minimum periodic production of standing snags per acre, turkey mast, forage, water, etc., or simultaneous multiple objectives such as net value. Model sizes are not greatly affected to the degree that the MAXMIN objective (MXN) is used to replace explicit constraints such as "nondeclining yield."

A second objective function (TMB) has been included for maximizing total timber production after setting a periodic harvest floor based on the initial MAXMIN solution in a rollover fashion. As it turns out, long-term sustained yield (LTSY) constrains the solution to this model in such a way that the rollover has no effect other than to find an alternate optimum. By relaxing the LTSY constraint the reader can see the effects of the initial MAXMIN solution more clearly. Experimentation with richer data sets is advised.

There are six key sections of input related to building the MAXMIN objective function and replacing the standard FORPLAN timber constraints. These have been marked in the data set by "*** **N** ***" where "N" is a numeral from 1 to 6. Explanations corresponding to each of these numbered sections are provided at the end of this appendix. An electronic copy of this data set as well as the current version of the FORPLAN Users Guide can be obtained from the WO-LMP Systems Section by telephoning (303) 498-1833.

```
::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::::
TIME
*YEARS1992 10 10 10 10
*YEAR GROUP 10
IDENTIFIERS
*LEVEL1         NOT IN USE
*LEVEL2         NOT IN USE
*LEVEL3         SLOPE CLASS
 <4     <40%         LT 40 PERCENT
*LEVEL4         WORK  GROUP
 LP     LOB  P       LOBLOLLY PINE
*LEVEL5         STOCKING
 GD     GOOD         GOOD STOCKING
*LEVEL6         CONDITION CLASS
 MA     MATURE       MATURE TIMBER
*LEVEL7         MANAGEMENT EMPHASIS
 TM     TIMBER       TIMBER MGT EMPHASIS
*LEVEL8         MANAGEMENT INTENSITY
 FF     FH—FH        STANDING VOL - FH: REGEN STAND - NATURAL REGEN & FINAL HARVEST
QUALIFIERS
 D      DIAM HARV  N O
TREATMENT TYPES
 C      CLEAR CT(EX)             Y Y Y N Y Y N Y N  Y  N
 D      CLEAR CT(RG)             Y Y Y N Y Y N Y Y  N  N
ACTIVITIES
 NONE   NO ACTIVITIES INCLUDED                      N C P        N N N
OUTPUTS AND ENVIRONMENTAL EFFECTS
 TMB    TMB: TIMBER VOLUMES           MCF           N C P        N N
 LMDA   LAMBDA: MAXMIN OPERATOR       10-MCF        N C P        N N           *** 1 ***

 INV    INV: INVENTORY VOLUME         MCF           N C Y        N N
 SAV    SAV: STAND AVERAGE VOL        MCF           N C Y        N N
 LTSY   LTSY: LONG-TERM SUS YLD       MCF           N C Y        N N
```

```
YIELD COMPOSITE NAMES FOR OUTPUTS/ACTIVITIES
#1TIMBY OUTPUTS RELATED TO TIMBER HARVEST
#2             TMB
#2INV      W TMB      TI T
#2SAV      W TMB      TI T
#2LTSY     W TMB      TI T
OBJECTIVE        MXN   1 N   N N    MAXMIN TIMBER VOLUME OBJ      *** 2 ***
 LMDA
OBJECTIVE        TMB   4 N   N N    TOTAL TIMBER VOLUME FOR FOUR DE-
CADES OBJ
 TMB
FOREST CONSTRAINTS
TIMBER HARVEST TMB
*LTSYC       LTSY
*VOLUME      SAV
*INVENTORY   INV
GN GENERAL RELATIONAL CONSTRAINTS
#1XN                      LMDA A P                                *** 3 ***
#2                                         1 1
#3                        TMB  A P
#4                                         1 4        10.
#1LSY                     LTSY A P                                *** 4 ***
#2                                         4 4
#3                        TMB  A P
#4                                         1 4                         1.
#1INV                      SAV  A P                               *** 5 ***
#2                                         4 4
#3                        INV  A P
#4                                         4 4        1.
AP PRESCRIPTION SOURCE INFORMATION
#1           LP
#2               TMFF        T1

SOURCE OF ACTIVITY/OUTPUTS
#1TIMB
#2               TM
HARVEST SOURCE
#1TMB
#2              LPGDMATMFF          4 8     3 4
```

```
YIELDS
AA 001       <4LPGDMA    100.
AA MXMN                  100.                       DUMMY AA FOR MAXMIN OBJ    *** 6 ***
RX SLAK                                                  SLACK RX FOR MAXMIN OBJ
RX MXMN                                                 ACTIVE RX FOR MAXMIN OBJ
                                  LMDAN        A 1.0
END OF DATA
ANALYSIS AREA INFORMATION
CODE TMB
*I           LPGDMATMFF
*A EXIST     04  LP MATURE TIMBER, GOOD STOCKING, EX: FH  REG: FH
        CA04   3.0   3.2   3.5   3.4
        CD04   8.5   9.0   9.5   9.7
*A REGEN     LP MATURE TIMBER, GOOD STOCKING, EX: FH  REG: FH
        DA01   0.6   1.5   2.4   3.2
        DD01   6.0   7.0   7.5   8.0
END OF DATA
```

---

[1] This line defines a FORPLAN output (LMDA) to be used as the MAXMIN operator. To reduce confusion it is recommended that it be defined with "per acre / per area" and "per year / per period" flags similar to those resource outputs being optimized (TMB in this case).

[2] Two lines define the MAXMIN objective function (MXN) and identify the MAXMIN operator (LMDA) as the output to be maximized. Note that a second objective has been provided on the following two lines to maximize timber production in a rollover run following the MAXMIN optimization.

[3] Four lines define a general relational constraint to ensure that each of the periodic timber harvest volumes is greater than or equal to **c** *times the MAXMIN operator. The value 10 could be any positive coefficient that reasonably scales the system. Note that since the MAXMIN operator is timeless, only one constraint row for that output needs to be defined. It is important, though, to ensure that the same time period (1 in this data set) is used for LMDA throughout the FORPLAN data set.*

[4] Four lines define a general relational constraint to keep periodic timber harvest volumes at or below long-term sustained yield capacity. Often such a constraint is invoked somewhat implicitly via a link to FORPLAN's standard nondeclining yield constraint. Since a nondeclining yield constraint would probably not be used with this MAXMIN objective, a more explicit constraint set must be defined if one wishes to keep harvests at or below long-term sustained yield capacity.

[5] Four lines define a general relational constraint to retain ending inventory greater than or equal to the average inventory projected among the prescriptions for future timber stands. This constraint is equivalent to FORPLAN's perpetual timber harvest constraint but allows more flexibility. By explicitly formulating this ending inventory constraint, the analyst may further qualify the nature of the constraint with additional input.

[6] Four lines define the "analysis area" and "prescriptions" needed to activate the MAXMIN operator as a FORPLAN output. The four lines do this by creating appropriate decision variables in the matrix. While other input options exist, the direct entry method shown here is probably the simplest. In this example, **M** *has been set to 100 "acres" and* **k** *has been set to 1 (LMDA produced in period 1 per "acre" of* **M** *allocated to prescription MXMN). Selection of these values (***c** *= 10,* **k** *= 1,* **M** *= 100) is predicated on knowing a priori that periodic timber harvest MCF in this model cannot exceed 1000, or* **ckM** *. With that mathematical product as an upper limit, the individual values may be set in any combination that seems appropriate.*

Appendix B

Sample FORPLAN Model Matrix Tableau

		1	1	G	1	1	1	1	1	1	G	G		R
				P	0	0	0	0	0	0	P	P		H
		M	S	X	0	0	0	0	0	0	L	I		S
		X	L	N	1	1	1	1	1	1	S	N		
		M	A		T	T	T	T	T	T	Y	V		
		N	K		1	1	1	1	1	1				
				1							4	4		
		1	1		1	2	3	4	5	6				
Maximize														
OB1MXN	1	1												
then														
OB2TMB	4				5.4	3.0	3.2	3.5	3.4					
subject to:														
AAMXMN		1	1										=	100.
A001					1	1	1	1	1	1			=	100.
GPXN	1R	1		-1									=	0.
GSXN	1R			-10.	3.0	3.0							≥	0.
GSXN	2R			-10.			3.2						≥	0.
GSXN	3R			-10.				3.5					≥	0.
GSXN	4R			-10.	2.4				3.4				≥	0.
GPLSY	4R				0.8	0.8	0.8	0.8	0.8	0.8	-1		=	0.
GSLSY	1R				3.0	3.0					-1		≤	0.
GSLSY	2R						3.2				-1		≤	0.
GSLSY	3R							3.5			-1		≤	0.
GSLSY	4R				2.4				3.4		-1		≤	0.
GPINV	4R				1.5	1.925	1.5	1.5	1.5	1.5		-1	=	0.
GSINV	4R				2.4	2.4	1.5	0.6	3.4	3.4		-1	≥	0.

Rows

OB1MXN 1	Objective function row for maximizing λ.
OB2TMB 4	Secondary objective function row for maximizing total timber harvest in all four periods after first maximizing λ. That activity level is then set as a constraint (using row "OB1MXN 1") before using this objective function. This procedure is is often called a "rollover run."
AAMXMN	Activity "constraint" row (RHS = $\boldsymbol{M}$ = 100.) for the MAXMIN "prescription" that produces λ.
AA001	Acreage constraint row for the resource land area (001).
GPXN 1R GSXN 1R... ...GSXN 4R	General relational constraint rows to ensure that periodic timber harvests (in the "GS" rows, one for each period) will be greater than or equal to some constant ($\boldsymbol{c}$ = 10.) times λ (in the "GP" row).
GPLSY 4R GSLSY 1R... ...GSLSY 4R	General relational constraint rows for the long-term sustained yield constraint. Long-term sustained yield coefficients are placed in the "GP" row and periodic timber harvest coefficients are placed in the "GS" rows (one for each period).
GPINV 4R GSINV 4R	General relational constraint rows for the ending inventory constraint. The average stand volume coefficients are placed in the "GP" row and the ending inventory coefficients are placed in the "GS" row.

Columns

1 MXMN 1	MAXMIN decision variable ($\boldsymbol{D}$). In this model it provides 1 unit ($\boldsymbol{k}$ = 1) of λ for every unit of activity in solution. FORPLAN treats this activity as "acres."
1 SLAK 1	Slack variable ($\boldsymbol{S}$) for the MAXMIN formulation.
GPXN 1	Transfer column ($\boldsymbol{T}$) that links the MAXMIN decision variable with the periodic timber harvests through general relational constraint rows.
1001T1 1... ...1001T1 6	Land resource decision variables ($\boldsymbol{X}_j$).
GPLSY 4	Transfer column for the general relational constraint that keeps periodic timber harvest at or below long-term sustained yield capacity.
GPINV 4	Transfer column for the general relational constraint that retains ending inventory greater than or equal to the average inventory.

U.S. Department of Agriculture
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Southwest

Great Plains

The Rocky Mountain Station is one of eight regional experiment stations, plus the Forest Products Laboratory and the Washington Office Staff, that make up the Forest Service research organization.

RESEARCH FOCUS

Research programs at the Rocky Mountain Station are coordinated with area universities and with other institutions. Many studies are conducted on a cooperative basis to accelerate solutions to problems involving range, water, wildlife and fish habitat, human and community development, timber, recreation, protection, and multiresource evaluation.

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Flagstaff, Arizona
Fort Collins, Colorado*
Laramie, Wyoming
Lincoln, Nebraska
Rapid City, South Dakota

*Station Headquarters: 240 W. Prospect Rd., Fort Collins, CO 80526